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How Can Brane Inflation Solve the Fine-Tuning Problem?

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Abstract Brane inflation can provide a promising framework for solving the fine-tuning problem in standard inflationary models. The aim of this paper is to illustrate the mechanism by which this can be achieved. By considering the supersymmetric two-stage inflation model we show that the initial fine-tuning of the coupling parameter can be controlled through a relation between the coupling parameter and the brane tension.

1 Introduction

Inflation has been initially invoked as a remedy for the conceptual problems of the cosmological hot big-bang model [1, 2]. These problems are essentially related to the fine-tuning of the initial conditions. However, inflation itself has given rise to another fine-tuning problem. Indeed, small values of the parameters of the theory are usually required in order to guarantee a sufficiently long period of inflation (i.e. flat potential) to solve the problems of the standard cosmological model (as the horizon and flatness ones), and to generate the correct magnitude of the density perturbations supposed to be the origin of the structure formation in the universe. This has been a great challenge for physicists.

It has been shown that this problem can be removed in the context of the hybrid inflation [3, 4] which is one of the most attractive models of inflation, and is particularly relevant to the supersymmetric realization of inflation [5] where a flat potential is naturally obtained. However, the problem reappears in some variants of hybrid inflation. Indeed, in reference [6] we have proposed a model of supersymmetric hybrid inflation which describes a scenario of two disconnected stages of inflation, but this can be only achieved with a severe fine-tuning of the coupling parameter. The same fine-tuning is required in order to generate the correct amplitude of the density perturbations.

Recently there has been a great deal of interest in conceiving our universe to be confined in a brane embedded in a higher dimensional space-time [7]. Such models are motivated

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by superstring theory solutions where matter fields (related to open string modes) live on the brane, while gravity (closed string modes) can propagate in the bulk [8–10]. In these scenarios extra dimensions need not be small [11, 12]. Another important consequence of these ideas is that the fundamental Planck scale M_{4+d} in $(4+d)$ dimensions can be considerably smaller than the effective Planck scale $M_p = 1.2 \times 10^{19}$ GeV in our four dimensional space-time.

It has also been noticed that in the context of extra dimensions and the brane world scenario the effective four-dimensional cosmology may deviate from the standard Big-Bang cosmology [13–17], which could lead to a different physics of the early universe. The origin of this difference is the correction to the Friedmann equation which becomes [14–17]

$$H^2 = \frac{8\pi}{3M_p^2}\rho\left[1 + \frac{\rho}{2\lambda}\right] \tag{1}$$

where λ is the brane tension. The new term in ρ^2 is dominant at high energies, but at low energies we must recover the standard cosmology, in particular during the nucleosynthesis, which leads to a lower bound on the value of the brane tension $\lambda \geq (1\text{ MeV})^4$.

With this modified Friedmann equation and within the slow-rolling paradigm the two slow-rolling parameters become [18]

$$\epsilon \equiv \frac{M_p^2}{16\pi}\left(\frac{V'}{V}\right)^2\left[\frac{2\lambda(2\lambda + 2V)}{(2\lambda + V)^2}\right], \tag{2}$$

$$\eta \equiv \frac{M_p^2}{8\pi}\left(\frac{V''}{V}\right)\left[\frac{2\lambda}{2\lambda + V}\right] \tag{3}$$

where, as usually assumed in the slow-roll approximation, the energy density is dominated by the potential energy V. It is clear that at low energies, $V \ll \lambda$, the two parameters reduce to the standard form [1], but at high energies, $V \gg \lambda$, this extra contribution to the Hubble expansion helps damp the rolling of the scalar field as ϵ and η are suppressed. Thus, *brane effects ease the condition for slow-roll inflation for a given potential* [18].

In the same way the number of e-folds during inflation become [18]

$$N \simeq -\frac{8\pi}{M_p^2}\int_{\sigma_1}^{\sigma_2} \frac{V}{V'}\left[1 + \frac{V}{2\lambda}\right] d\sigma. \tag{4}$$

Similarly, at high energies this expression yields more inflation between any two values of the inflaton for a given potential. It follows from these two remarks that we can hope to solve or to reduce the fine-tuning problem in the inflationary models in the context of the braneworld scenario.

The possibility of solving the fine-tuning problem in extra dimensions theories has also been studied in reference [19] where the authors have proposed a generalization of the so-called assisted inflation [20] (see also [21]) for power law potentials where the Kaluza–Klein modes of the 5-dimensional scalar field constitute the source of the necessary multiplicity of scalar fields. In this model the 4-dimensional coupling constant of the K-K fields is determined by the 5-dimensional one divided by the number of the K-K modes. As a result the effective coupling constant is suppressed by the number of scalar fields present in the theory, which means that it becomes naturally small (to satisfy the COBE constraint) without the need of any fine-tuning of the fundamental coupling constants.

Our purpose in this paper is to illustrate a mechanism by which this problem can be solved in the braneworld picture. Indeed, the cosmological constraint gives rise to a relation between the brane tension and the coupling parameter through which the latter can take a natural value while the value of the former remains acceptable.

2 The Initial Model

In Ref. [6] we have contributed to the resolution of the problem encountered in the first realization of the hybrid inflation model in supersymmetry: the generation of a slope for the scalar potential. Indeed, the initial version of the supersymmetric hybrid inflation was based on a superpotential of the form [22, 23]

$$W = \kappa S(-\mu^2 + \bar{\phi}\phi) \tag{5}$$

where $\bar{\phi}$, ϕ is a conjugate pair of superfields transforming as non-trivial representation of some gauge group G under which the superfield S is neutrally charged, the coupling parameter κ and the mass scale μ can be taken to be real and positive.

Such a superpotential gives a scalar potential which is identical to the potential of the initial non-supersymmetric hybrid model [3, 4] but without the mass-term of the inflaton field. Such a mass-term is necessary for inflation since it gives the slope of the valley of minima, and then drives the inflaton to its critical value where inflation ends.

Our approach was based on the modified superpotential [6]

$$W = \kappa S(-\mu^2 + \bar{\phi}\phi) + \frac{\lambda_S}{3} S^3. \tag{6}$$

From the above superpotential we have derived an effective potential of the form

$$V(\varphi, \sigma) = \kappa^2 \left(\mu^2 - \frac{\varphi^2}{4}\right)^2 + g^2 \frac{\varphi^2 \sigma^2}{4} + \lambda_\sigma \frac{\sigma^4}{4} \tag{7}$$

(where $\lambda_\sigma = \lambda_S^2$) which is the same as the one of the initial Linde's model but the mass-term has been replaced by a quartic self-coupling one.[1]

In the resulting model a first phase of inflation occurs where the potential is dominated by the σ^4-term, and ends when the inflaton reaches a value $\sigma_e \sim \mathcal{O}(M_p)$ ($\epsilon(\sigma_e) \sim 1$) which is much greater than $\sigma_c = \sqrt{2}\kappa\mu/g$. The inflaton undergoes a phase of oscillations about its minimum during which its energy decreases until the vacuum energy $V = \kappa^2\mu^4$ becomes dominant, then a second stage of inflation begins. The achievement of such a scenario requires

$$\lambda_\sigma \simeq 5 \times 10^{-12}. \tag{8}$$

3 The Brane Version

This model describes a cosmologically rich scenario since the first inflationary phase may produce the homogeneity beyond the Hubble radius which makes natural the onset of the

[1] The scalar fields φ and σ are proportional to the real parts of the scalar components of the superfields $\bar{\phi}$, ϕ and S respectively.

second stage during which the interesting length scales leave the horizon. However, a severe fine-tuning of the self-coupling constant is required to achieve such a second stage and to generate the correct magnitude of the spectrum of density perturbations.

The above result may be interpreted as reflecting the simple fact that the two-stage scenario is less favorable in the standard inflationary cosmology. As it was explained in the introduction this situation can be improved in the context of the braneworld scenario where inflation occurs more easily [see (2), (3)].

In this section we shall restrict ourselves to the simple picture of the so-called Randall–Sundrum II model [24] where the matter of the universe is confined in a (3 + 1) brane with positive tension (the bulk space being empty), and the higher dimensional space is non-compact. The fundamental equations in this case are (1–4).

There are two different ways in which inflation may end:

(i) When σ reaches the critical value $\sigma_c = \sqrt{2}\kappa\mu/g$ inflation ends by instability of the field ϕ and σ drops towards the global minimum of the potential.
(ii) If the slow-rolling conditions are no longer valid ($\epsilon, \eta \sim O(1)$). This may occur at some value $\sigma > \sigma_c$.

In the two-stage inflationary model the first stage occurs at large values of σ when the quartic term is dominant, and ends in the second way. In the present case the two slow-rolling parameters (2) and (3) are related by the relation $\epsilon = 4/3\eta$, and then the condition for the end of inflation is reduced to $\epsilon \sim O(1)$, the corresponding value of σ is given by

$$\sigma_e^6 = \frac{16M_p^2}{\pi}\frac{\lambda}{\lambda_\sigma}. \tag{9}$$

When the energy of the inflaton field decreases and the vacuum term becomes dominant the second stage of inflation begins. Assuming that in the brane inflationary models (as in the standard ones) the relevant scales leave the horizon about 60 e-foldings before the end of inflation, we shall be interested in the inflaton values $\sigma < \sigma_{60}$ where σ_{60} is the corresponding value. Hence, the condition for the second stage of inflation to take place is:

$$\frac{\lambda\sigma_{60}^4}{4} \ll \kappa^2\mu^4. \tag{10}$$

The value of σ_{60} is determined in our model by the equation:

$$N \simeq -\frac{8\pi}{M_p^2}\int_{\sigma_{60}}^{\sigma_c}\frac{V^2}{2\lambda V'}d\sigma \tag{11}$$

which gives

$$\frac{1}{\sigma_{60}^2} \simeq \frac{g^2}{2\kappa^2\mu^2} - \frac{30M_p^2\lambda_\sigma\lambda}{\pi\kappa^4\mu^8}. \tag{12}$$

The condition (10) then becomes

$$\pi\kappa^2\mu^6 g^2 \gg 60M_2^2\lambda_\sigma\lambda. \tag{13}$$

In the brane inflation context the main cosmological constraint comes from the amplitude of the scalar perturbations which is given by [18]:

$$A_s^2 \simeq \left(\frac{512\pi}{75M_p^6}\right)\frac{V^3}{V'^2}\left[\frac{2\lambda + V}{2\lambda}\right]^3\Bigg|_{k=aH}. \tag{14}$$

In our model we obtain

$$A_s^2 = \frac{8\pi(\pi\kappa^2\mu^6 g^2 - 60M_p^2\lambda_\sigma\lambda)^3}{75M_p^6\lambda_\sigma^2\lambda^3\pi^3}. \tag{15}$$

The COBE observed value of A_s is

$$A_s = 2\times 10^{-5} \tag{16}$$

implies

$$\frac{8\pi}{75\lambda_\sigma^2\lambda^3}\left(\frac{\kappa^2\mu^6 g^2}{M_p^2}\right)^3 = 4\times 10^{-10} \tag{17}$$

where we have used the condition (13) to neglect small terms.

If we take the same values of the different parameters as in Ref. [6] (i.e. $\mu \simeq 10^{15}$ GeV and $\kappa, g \simeq 10^{-1}$) a value of $\lambda_\sigma \simeq 10^{-1}$ is obtained by taking $\lambda \simeq (10^{12}\text{ GeV})^4$. This value of the brane tension is compatible with the upper bound [25]:

$$\lambda \le (7\times 10^{14}\text{ GeV})^4 \tag{18}$$

and ensures that the potential energy remains dominant in the early Universe: $\kappa^2\mu^4/2\lambda \simeq 10^{10}$.

In this model the scale dependence of the scalar perturbations which is given by the spectral tilt

$$n_s \equiv \frac{d\ln A_s^2}{d\ln k} = 2\eta - 6\epsilon \tag{19}$$

has the value

$$n_s = 0.9965. \tag{20}$$

Furthermore, the brane tension is involved in the relation between the four and the five dimensional Planck mass [26]

$$M_p = \sqrt{\frac{3}{4\pi}}\frac{M_5^3}{\sqrt{\lambda}} \tag{21}$$

which gives in this case

$$M_5 = 0.6\times 10^{14}\text{ GeV}. \tag{22}$$

This value is also compatible with the upper bound [26]

$$M_5 \le 2\times 10^{-3}M_p. \tag{23}$$

At this point it is interesting to examine the case of a simple hybrid inflation model where the inflaton energy remains dominant until the instability point σ_c. In this case the relevant value of the inflaton field is:

$$\sigma_{60}^6 = 1440\frac{M_p^2\lambda}{\pi\lambda_\sigma}. \tag{24}$$

The amplitude of the scalar perturbations then becomes:

$$A_s^2 = \frac{284^3}{75\pi^2}\lambda_\sigma \tag{25}$$

which is independent of the brane tension. The constraint (16) yields

$$\lambda \simeq 10^{-14}. \tag{26}$$

This result can be interpreted as a consequence of the absence of any relation between the brane tension and the coupling parameter through which one can control this fine tuning, we then recover the result of the standard case [27].

In this paper the model of supersymmetric two-stage inflation has been reexamined in the context of the braneworld scenario. The aim of this extension was to relax the fine-tuning of the coupling parameter in the initial version. This goal has been achieved through a relation between the coupling parameter and the brane tension. This relation allows us to control the fine tuning of the parameter while keeping the value of the brane tension in the right bounds.

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References

1. Kolb, E.W., Turner, M.S.: The Early Universe. Addison–Wesley, Reading (1990)
2. Linde, A.D.: Particle Physics and Inflationary Cosmology. Harwood, Reading (1990)
3. Linde, A.D.: Phys. Lett. B **259**, 38 (1991)
4. Linde, A.D.: Phys. Rev. D **49**, 748 (1994)
5. Lyth, D.H., Riotto, A.: Phys. Rep. **314**, 1 (1999)
6. Boutaleb, H., Chafik, A., Marrakchi, A.L.: Acta Phys. Slovaca **51**, 297 (2001)
7. Polchinski, J.: String Theory. Cambridge University Press, Cambridge (1998)
8. Polchinski, J.: Phys. Rev. Lett. **75**, 4724 (1995)
9. Horava, P., Witten, E.: Nucl. Phys. B **460**, 506 (1996)
10. Lukas, A., Ovrut, B.A., Waldram, D.: Phys. Rev. D **60**, 086001 (1999)
11. Antoniadis, I.: Phys. Lett. B **264**, 317 (1990)
12. Arkani-Hamed, N., Dinopoulos, S., Dvali, G.: Phys. Lett. B **429**, 263 (1998)
13. Lukas, A., Ovrut, B., Waldram, D.: Phys. Rev. D **61**, 023506 (2000)
14. Binetruy, P.: Nucl. Phys. B **565**, 269 (2000)
15. Binetruy, P., Deffayet, C., Ellwanger, U., Langlois, D.: Phys. Lett. B **477**, 269 (2000)
16. Shiromizu, T., Maeda, K., Sasaki, M.: Phys. Rev. D **62**, 024012 (2000)
17. Flangan, E.E., Tye, S.H., Wasserman, I.: Phys. Rev. D **62**, 044039 (2000)
18. Maartens, B., Wands, D., Basset, B.A., Heart, I.P.C.: Phys. Rev. D **62**, 041301 (2000)
19. Kanti, P., Olive, K.A.: Phys. Rev. D **60**, 043502 (1999)
20. Liddle, A.D., Mazumdar, A., Svhunck, F.E.: Phys. Rev. D **58**, 061301 (1998)
21. Malik, K.A., Wands, D.: Phys. Rev. D **59**, 123501 (1999)
22. Dvali, G., Shafi, Q., Shaefer, R.: Phys. Rev. Lett. **73**, 1886 (1994)
23. Copeland, E.J., Liddle, A.R., Lyth, D.H., Stewart, E.D., Wands, D.: Phys. Rev. D **49**, 6410 (1994)
24. Randall, L., Sundrum, R.: Phys. Rev. Lett. **83**, 4690 (1999)
25. Shiromizu, T., Maeda, K., Sasaki, M.: Phys. Rev. D **62**, 024012 (2000)
26. Boutaleb, H., Chafik, A., Marrakchi, A.L.: Phys. Lett. B **574**, 89 (2003)
27. Roberts, D., Liddle, A.D., Lyth, D.: Phys. Rev. D **51**, 4122 (1995)